
Dynamic Response of Linear Viscoelastic Bodies under Periodic Stresses and Strains

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Abstract (12pt)

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The generalized linear differential equation with constant coefficients has been considered for finding the non-dimensional form of constitution equation of standard linear solid viscoelastic model depending upon the time dependent parameter τ and frequency ω . The non-dimensional form has been derived by taking the sinusoidal variation of stress ($\sigma = \sigma_0 \sin \omega t$ or $P = P_0 \sin \omega \theta$) and strain $\gamma = \gamma_0 \sin \omega \theta$. The viscoelastic body's behavior has also been discussed at different values of τ and results are derived at low frequency and at high frequency and it has been concluded that linear viscoelastic bodies behave periodically under the influence of periodic stress and strains.

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1. Introduction

Elastic solids and viscous fluids differ widely in their deformational characteristics. Elasticity deformed bodies return to a natural or undeformed state upon removal of applied loads. Viscous

fluids, however possesses no tendency at all for deformational recovery. Also elastic stress is related directly to deformation whereas stresses in viscous fluids depends (except for hydrostatic component) upon rate of deformation. Material behavior which incorporates a blend of both elastic and viscous characteristic is referred to as viscoelastic behavior. The elastic (Hookean) solids and viscous (Newtonian) fluid represent opposite endpoints of a wide spectrum of viscoelastic behavior. Linear viscoelasticity may be introduced conveniently from a one dimensional view point through a discussion of mechanical models which portray the deformation response of various viscoelastic materials. The mechanical elements of such models are the massless linear spring with spring constant G and the viscous dashpot having a viscosity constant η . Kakar.R. K.Kaur and K.C.Gupta (2012) has study the rheological responses of five-parameter viscoelastic models under dynamic loading. This model is chosen for studying elastic, viscous, and retarded elastic responses to shearing stress. Simo (1987), Holzapfel and Reiter (1995), Holzapfel (1996), Simo and Hughes (1998) have proposed alternate models in which the evaluation of viscous non-linear stresses is defined by non-linear differential equation that mimics the force relaxation process taking place in linear rheological models. J.Bonet (2001) discussed the large strain constitutive viscous model. These are based on particular linear relaxation form of the generalized Maxwell model which leads to viscoelastic formulation that can be seen as a particular case of large strain viscoelastic model based on maximum plastic dissipation. S.Imray (1953) also studied dynamic behaviour of linear rheological bodies under periodic stresses.

In this paper, we consider the standard linear solid model (Kelvin unit in series with a spring) and discuss its stress-strain relation in the Non-Dimensional form and find its solution in non-dimensional form. Expressing the time and stress in non-dimensional forms, a universal equation is obtained, depending upon a single non-dimensional parameter, the time factor $\tau = \frac{R}{L} = \frac{G}{H}$, where G is asymptotic rigidity or static elastic modulus, H is elastic firmness or dynamic modulus, R is the time of relaxation and L is of lagging or retardation.

2. Stress-strain relation in the Non-Dimensional Form

The series combination of a linear spring and Kelvin model is known as Standard Linear Solid model as shown in Fig.1.

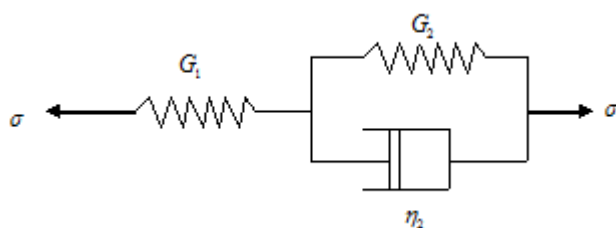


Fig. 1 (Standard Linear Solid)

The stress-strain relation is [2] $\left(D + \frac{G_1 + G_2}{\eta_2}\right) \sigma = \left(G_1 D + \frac{G_1 G_2}{\eta_2}\right) \gamma$

$$\text{Or} \quad (G_1 + G_2)\sigma + \eta_2\dot{\sigma} = G_1G_2\gamma + G_1\eta_2\dot{\gamma} \quad (1)$$

On comparing the equation (1) with the rheological equation of a generalized homogeneous isotropic linear body with constant rheological coefficients a_1, a_2, a_3, a_4 relating to the four deviators of stress σ , strain γ , and their time rates $\dot{\sigma} = \frac{\partial\sigma}{\partial t}$ and $\dot{\gamma} = \frac{\partial\gamma}{\partial t}$ given by equation (2)[8]

$$a_1\sigma + a_2\dot{\sigma} = a_3\gamma + a_4\dot{\gamma} \quad (2)$$

we get

$$a_1 = G_1 + G_2, a_2 = \eta_2, a_3 = G_1G_2, a_4 = G_1\eta_2$$

Then we define

$$G = \frac{a_3}{a_1} = \frac{G_1G_2}{G_1+G_2} = \left(\frac{1}{G_1} + \frac{1}{G_2}\right)^{-1} = (\text{Dim } FL^{-2}) = G_1 \left(1 + \frac{G_2}{G_1}\right)^{-1} = G_2 \left(1 + \frac{G_1}{G_2}\right)^{-1} \quad 3(a)$$

= static modulus of elasticity / Asymptotic rigidity, for $\dot{\gamma} = \dot{\sigma} = 0$.

$$H = \frac{a_4}{a_2} = G_1 = (\text{Dim: } FL^{-2}) = \text{Dynamic modulus of elastic firmness.} \quad 3(b)$$

$$R = \frac{a_2}{a_1} = (\text{Dim: } T) = \text{Time of relaxation coefficient of stress rate} = \frac{\eta_2}{G_1+G_2}. \quad 3(c)$$

$$L = \frac{a_4}{a_3} = (\text{Dim: } T) = \text{Time of lagging coefficient of strain} = \frac{\eta_2}{G_2}. \quad 3(d)$$

$$\mu = \frac{a_4}{a_1} = (\text{Dim: } FL^{-2}T) = \text{solid viscosity} = \frac{G_1\eta_2}{G_1+G_2} = \left(1 + \frac{G_2}{G_1}\right)^{-1} \eta_2. \quad 3(e)$$

$$\eta = \frac{a_2}{a_3} = (\text{Dim: } F^{-1}L^2T) = \text{endosity} = \frac{\eta_2}{G_1G_2} = \frac{\eta_2}{G_2} \frac{1}{G_1} = \frac{L}{G_1}. \quad 3(f)$$

$$\tau = \frac{a_2a_3}{a_1a_4} = \frac{\eta_2G_1G_2}{(G_1+G_2)G_1\eta_2} = \frac{G_2}{G_1+G_2} = (\text{no dim}) = \text{Time factor} \quad 3(g)$$

As only these coefficients are independent, we have the following relations

$$G = \frac{R}{\eta} = \frac{\mu}{L} = \frac{G_1G_2}{G_1+G_2} = G_1 \left(1 + \frac{G_2}{G_1}\right)^{-1} = G_2 \left(1 + \frac{G_1}{G_2}\right)^{-1} \quad 4(a)$$

$$H = \frac{L}{\eta} = \frac{\mu}{R} = G_1 \quad 4(b)$$

$$R = \frac{\mu}{H} = \frac{G}{\eta} = \frac{\eta_2}{(G_1+G_2)} = \left(\frac{G_1}{G_2} + \frac{G_2}{\eta_2}\right)^{-1} \quad 4(c)$$

$$L = \frac{\mu}{G} = H\eta = \frac{G_2}{\eta_2} \quad 4(d)$$

$$\mu = GL = RH = \frac{\eta_2 G_1}{(G_1 + G_2)} = \eta_2 \left(1 + \frac{G_1}{G_2}\right)^{-1} \quad 4(e)$$

$$\eta = \frac{R}{G} = \frac{L}{H} = \frac{\eta_2}{G_1 G_2} \quad 4(f)$$

$$\tau = \frac{R}{L} = \frac{G}{H} = \frac{G_2}{G_1 + G_2} = \left(1 + \frac{G_1}{G_2}\right)^{-1} \quad 4(g)$$

Solution of equation (2) If starting from rest i.e. $\gamma = 0, \sigma = 0$, we apply strain $\gamma(t)$ which is being a ratio in the non-dimensional form, the equation (2) can be rewritten as:

$$\frac{a_1}{a_2} \sigma + \dot{\sigma} = \frac{a_3}{a_2} \gamma + \frac{a_4}{a_2} \dot{\gamma}$$

$$\text{Or} \quad \dot{\sigma} + \frac{1}{R} \sigma = \frac{1}{\eta} \gamma + H \dot{\gamma} \quad (5)$$

And its solution is

$$\sigma e^{\int \frac{1}{R} dt} = \frac{1}{\eta} \int_0^t \left(\frac{1}{\eta} \gamma + H \dot{\gamma}\right) e^{\int \frac{1}{R} dt} dt$$

$$\text{Or} \quad \sigma e^{\frac{t}{R}} = \frac{1}{\eta} \int_0^t \gamma e^{\frac{t}{R}} dt + H \int_0^t \dot{\gamma} e^{\frac{t}{R}} dt$$

$$\text{Or} \quad \sigma e^{\frac{t}{R}} = \frac{1}{\eta} \int_0^t \gamma e^{\frac{t}{R}} dt + H \left\{ \left(\gamma e^{\frac{t}{R}}\right)_t - \int_0^t \frac{\gamma}{R} e^{\frac{t}{R}} dt \right\}$$

$$\sigma e^{\frac{t}{R}} = H \gamma e^{\frac{t}{R}} + \left(\frac{1}{\eta} - \frac{H}{R}\right) \int_0^t \gamma e^{\frac{t}{R}} dt$$

$$\sigma = H \gamma + \left(\frac{1}{\eta} - \frac{H}{R}\right) e^{-\frac{t}{R}} \int_0^t \gamma e^{\frac{t}{R}} dt$$

$$\sigma = \frac{G}{\tau} \gamma + \left(\frac{G}{R} - \frac{H}{R}\right) e^{-\frac{t}{R}} \int_0^t \gamma e^{\frac{t}{R}} dt$$

Taking $\theta = t/R, R d\theta = dt$

$$\frac{\sigma}{G} = \frac{\gamma}{\tau} + \left(1 - \frac{1}{\tau}\right) e^{-\theta} \int_0^{\theta} \gamma(\theta') e^{\theta'} d\theta'$$

And by taking $P = \frac{\sigma}{G}$ i.e. non-dimensional form of stress, the above equation can be written as

$$P = \frac{\gamma}{\tau} + \left(1 - \frac{1}{\tau}\right) e^{-\theta} \int_0^{\theta} \gamma(\theta') \exp(\theta') d\theta'$$

Or
$$\tau P = \gamma - (1 - \tau) e^{-\theta} \int_0^{\theta} \gamma(\theta') \exp(\theta') d\theta' \quad (6)$$

Which is the required non-dimensional form of solution of equation (2).

To determine deformation/strain under time varying stress: Starting from equilibrium/rest position ($\gamma = 0, \sigma = 0$ or $P = 0$ at $t = 0$) and applying time dependent stress $\sigma(t)$ or numerical stress $P(\theta)$.

General equation of rheology in the differential form for the model having element (springs/Dashpot) is expressed as

$$a_3 \gamma + a_4 \dot{\gamma} = a_1 \sigma + a_2 \dot{\sigma} \quad (7)$$

Or

$$\dot{\gamma} + \frac{a_3}{a_4} \gamma = \frac{a_1}{a_4} \sigma + \frac{a_2}{a_4} \dot{\sigma}$$

Or

$$\dot{\gamma} + \frac{1}{L} \gamma = \frac{1}{\mu} \sigma + \frac{1}{H} \dot{\sigma} \quad (8)$$

Solution: Using $\mu = LG$ and considering eq.(8) as a linear equation in γ , we get

$$\gamma = \frac{\sigma(t)}{H} + \frac{1}{L} \left(\frac{1}{G} - \frac{1}{H} \right) e^{-\frac{t}{L}} \int_0^t \sigma(t') e^{\frac{t'}{L}} dt'$$

Or

$$\gamma = \frac{\sigma(t)}{H} + \frac{1}{L} \left(\frac{1}{G} - \frac{1}{H} \right) e^{-\frac{t}{L}} \int_0^t \sigma(t') e^{\frac{t'}{L}} dt'$$

$$\gamma = \frac{1}{G} \cdot \frac{G}{H} \sigma(t) + \frac{1}{L} (1 - \tau) e^{-\frac{t}{L}} \int_0^t \sigma(t') e^{\frac{t'}{L}} dt' \quad (9)$$

Transforming to non-dimensional form by defining $P = \frac{\sigma}{G}$, $\theta = \frac{t}{L}$, $dt = L d\theta$ and $\tau = \frac{R}{L} = \frac{G}{H}$, we express the equation (9) as

$$\gamma = \tau P(\theta) + (1 - \tau) e^{-\theta} \int_0^{\theta} P(\theta') e^{\theta'} d\theta' \quad (10)$$

Which is the required equation of deformation for given stress $P(\theta)$.

3. Periodic Stress

When the stress is applied in sinusoidal form as $\sigma = \sigma_0 \sin \omega t$ or $P = P_0 \sin \omega \theta$ where $f = \frac{2\pi}{T}$ is the frequency, $\omega = Lf = \frac{2\pi L}{T}$ is the numerical frequency, T is the time period, σ_0 is the stress amplitude, $P_0 = \frac{\sigma_0}{G}$ is the numerical stress amplitude, in equation (10), we get

$$\gamma = P_0 \left\{ \tau \sin \omega t + (1 - \tau) e^{-\theta} \int_0^{\theta} \sin \omega \theta' \exp(\omega \theta') d\theta' \right\}$$

Or

$$\gamma = P_0 \{ \tau \sin \omega t + (1 - \tau) e^{-\theta} I \} \quad (11) \text{ where}$$

$I = \int_0^{\theta} \sin \omega \theta' \exp(\omega \theta') d\theta'$ and on solving it integral by parts, we get

$$I = \left(\frac{\sin \omega \theta - \omega \cos \omega \theta}{1 + \omega^2} \right) e^{\theta} + \frac{\omega}{1 + \omega^2}$$

Using the value of I in equation (11), we get

$$\gamma = \frac{P_0}{1 + \omega^2} [\{ (1 + \tau \omega^2) \sin \omega \theta - (1 - \tau) \omega \cos \omega \theta \} + (1 - \tau) \omega e^{-\theta}] \quad (12)$$

Taking $1 + \tau \omega^2 = r \cos \alpha$, $(1 - \tau) \omega = r \sin \alpha$, where $r^2 = (1 + \omega^2)(1 + \tau^2 \omega^2)$ and $\tan \alpha = (1 - \tau) \omega / (1 + \tau \omega^2)$ in equation (12), we get

$$\gamma = P_0 \varphi \{ \sin(\omega \theta - \alpha) + \sin \alpha e^{-\theta} \} = \gamma_0 \{ \sin(\omega \theta - \alpha) + \sin \alpha e^{-\theta} \} \quad (13)$$

$$\text{where } \gamma_0 = P_0 \varphi \text{ and } \varphi = \sqrt{\frac{1 + \tau^2 \omega^2}{1 + \omega^2}} \text{ and } \frac{\sigma_0}{\gamma_0} = G \sqrt{\frac{1 + \tau^2 \omega^2}{1 + \omega^2}} = \tan \delta \text{ (say)} \quad (14)$$

In equation (13), the exponential term vanishes when t or $\theta \rightarrow \infty$ and we get finally a pure sinusoidal strain with amplitude γ_0 and phase difference α .

Here we see that both α and φ depends upon τ and numerical frequency ω .

$$\text{when } \tau = 0, \varphi = \sqrt{\frac{1}{1 + \omega^2}}$$

$$\text{when } \tau \geq 0, \varphi \geq 1$$

$$\text{and for } \tau = \infty, \varphi \rightarrow \infty.$$

$$\text{At low frequencies i.e. when } \omega \rightarrow 0, \varphi \rightarrow 1 \text{ and high frequencies } \omega \rightarrow \infty, \varphi \rightarrow \tau. \quad (15)$$

$$\text{Also when } \omega \rightarrow 0, \tan \delta \rightarrow G = G_0 \text{ and when } \omega \rightarrow \infty, \tan \delta \rightarrow \frac{G}{\tau} = H = G.$$

4. Periodic Strain

Taking $\gamma = \gamma_0 \sin \omega \theta$ i.e. varying the strain periodically in equation (6), we get

$$\tau P = \frac{\gamma_0}{1+\omega^2} \{(\tau + \omega^2)\sin\omega\theta + \omega(1 - \tau)\cos\omega\theta - (1 - \tau)\omega e^{-\theta}\} \quad (16)$$

Taking $\tau + \omega^2 = r\cos\alpha$, $\omega(1 - \tau) = r\sin\alpha$ so that $r^2 = (\tau^2 + \omega^2)(1 + \omega^2)$ and $\tan\alpha = \frac{(1-\tau)\omega}{\tau+\omega^2}$, the equation (16) can be expressed as

$$\tau P = P_0 \{\sin(\omega\theta + \alpha) - \sin\alpha e^{-\theta}\} = P_0 \left\{ \sin\left(\frac{\omega}{L}t + \alpha\right) - \sin\alpha e^{-\frac{t}{L}} \right\} \quad (17)$$

$$\text{Where } P_0 = \gamma_0\varphi \text{ and } \varphi = \sqrt{\frac{\tau^2 + \omega^2}{(1 + \omega^2)}} \quad (18)$$

In equation (18), on taking t or $\theta \rightarrow \infty$, the exponential term vanishes and we find pure sinusoidal stress with amplitude $P_0 = \gamma_0\varphi$ and phase difference α .

$$\text{when } \tau = 0, \varphi = \omega \sqrt{\frac{1}{1 + \omega^2}}$$

$$\text{when } \tau \geq 0, \varphi \geq 1$$

and for $\tau = \infty, \varphi \rightarrow \infty$.

Also for low frequencies when $\omega \rightarrow 0, \varphi \rightarrow \tau\varphi \rightarrow 1$ and high frequencies $\omega \rightarrow \infty, \varphi \rightarrow 1$. So, it is clear from the above results and the result given by equation (15) that both the viscoelastic bodies behave periodically under the influence of sinusoidal stress and sinusoidal strain.

5. Stress-Strain Conic Representation

$$\text{We know Stress} \quad P = P_0 \sin\omega\theta \quad (19)$$

$$\text{and strain/deformation} \quad \gamma = \gamma_0 \{\sin(\omega\theta - \alpha) + e^{-\theta} \sin\alpha\} \quad (20)$$

$$\text{Let } A = \left(\frac{\gamma}{\gamma_0} - e^{-\theta} \sin\alpha\right) = \sin(\omega\theta - \alpha) = \sin\omega\theta \cos\alpha - \cos\omega\theta \sin\alpha \quad (21)$$

Using equations (19) and equation (20) in equation (21), we have the relation

$$\sin^2\alpha = \left(\frac{P}{P_0}\right)^2 - 2\left(\frac{\gamma}{\gamma_0} - e^{-\theta} \sin\alpha\right) \frac{P}{P_0} \cos\alpha + \left(\frac{\gamma}{\gamma_0} - e^{-\theta} \sin\alpha\right)^2 \quad (22)$$

Which is the required equation of conic with co-ordinate axes $\frac{P}{P_0}$ and $\frac{\gamma}{\gamma_0}$ and centre at $(e^{-\theta} \sin\alpha, 0)$ on the $\frac{\gamma}{\gamma_0}$ axis which varies with t or θ . For t or $\theta \rightarrow 0, (\sin\alpha, 0)$ and t or $\theta \rightarrow \infty, (0, 0)$, the exponential term vanishes in equation (22) and we get

$$\left(\frac{P}{P_0}\right)^2 - 2\frac{P}{P_0} \frac{\gamma}{\gamma_0} \cos\alpha + \left(\frac{\gamma}{\gamma_0}\right)^2 = \sin^2\alpha \quad (23)$$

Which is the equation of ellipse with axis along $\frac{\gamma}{\gamma_0}$ and $\frac{P}{P_0}$.

Inclination β (say) of major axis of the ellipse with $\frac{\gamma}{\gamma_0}$ axis is given by

$$\tan 2\beta = -2 \frac{\cos \alpha}{1 - \cos \alpha} = \infty = \tan \frac{\pi}{2} \quad \text{or} \quad \beta = \frac{\pi}{4}.$$

Hence axis of the ellipse bisect the axis of the coordinate along $\frac{\gamma}{\gamma_0}$ and $\frac{P}{P_0}$. If we take semi major axis of ellipse as ' a ' and semi minor axis as ' b ' then vertices of the major axis of the ellipse are $(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}), (\frac{-a}{\sqrt{2}}, \frac{-a}{\sqrt{2}})$ and that minor axis are $(\frac{-b}{\sqrt{2}}, \frac{b}{\sqrt{2}}), (\frac{b}{\sqrt{2}}, \frac{-b}{\sqrt{2}})$. Using equation (23) we obtained $a = \sqrt{2} \cos \alpha$ and $b = \sqrt{2} \sin \alpha$, where $\sin \alpha = \frac{(1-\tau)\omega}{\sqrt{(1+\omega^2)(1+\tau^2\omega^2)}}$ and $\cos \alpha = \frac{(1+\tau\omega^2)}{\sqrt{(1+\omega^2)(1+\tau^2\omega^2)}}$.

For $\tau = 1$ and $\alpha = 0$, we get $b = 0$, $a = \sqrt{2}$ i.e. ellipse reduces to a straight line along $\frac{\gamma}{\gamma_0}$ axis.

For $\tau > 1$ and $b < a$, major axis of the ellipse lies along the line with slope 1 or inclination with $\frac{\gamma}{\gamma_0}$ is $\frac{\pi}{4}$ and minor axis of the ellipse is inclined to $\frac{\gamma}{\gamma_0}$ at $\frac{3\pi}{4}$.

For $\tau < 1$ and $a < b$, major axis of the ellipse lies along the line with inclination is $\frac{3\pi}{4}$ and minor axis of the ellipse is along the line with inclination to $\frac{\pi}{4}$.

So, it is concluded that the direction of the major and minor axis of the ellipse interchange according to $\tau > 1$ or $\tau < 1$ i.e. depends upon G and H .

In case of dimensional stress – strain (σ, γ) , if the angle of inclination of the a -axis of the ellipse with γ axis is δ . We have $\tan \delta = \frac{G}{\phi} = \sqrt{\frac{1+\tau^2\omega^2}{1+\omega^2}} \geq G$ for $\tau \geq 1$ and for $\omega \rightarrow 0$, $\tan \delta \rightarrow G$ and when $\omega \rightarrow \infty$, $\tan \delta \rightarrow \frac{G}{\tau} = H$, where G and H are elastic modullii; G is the static modulus valid for constant and low frequency stresses and H is the dynamic modulus valid for high frequency stresses. This may serve as basis for the experimental determination of G and H or τ of a linear viscoelastic body.

6. Results and Conclusion

When the stress is applied in sinusoidal form as $\sigma = \sigma_0 \sin ft$ or $P = P_0 \sin \omega \theta$, we find that deformation in linear viscoelastic body is also vary periodically i.e. $\gamma = \gamma_0 \sin(\omega \theta - \alpha)$ with amplitude γ_0 and phase difference α , when t or $\theta \rightarrow \infty$ and when body is strained by taking strain varying periodically i.e. $\gamma = \gamma_0 \sin \omega \theta$, we found pure sinusoidal stree with amplitude $P_0 = \gamma_0 \phi$ and phase difference α . By representing the stress -strain variation in conic form we see that the curve tends to asymptotically towards an ellipse and the direction of the major and minor axis of the ellipse interchange according to $\tau > 1$ or $\tau < 1$ i.e. depends upon G and H , where G (static modulus) valid for constant and low frequency stresses and H (dynamic modulus) valid for high frequency stresses.

The results obtained in non-dimensional form of linear viscoelastic models also used to study the soil behavior by choosing appropriate parameters. Further, damped mass spring Voigt model, a Maxwell model and choosing a suitable three parameter viscoelastic models etc. can be used to develop the mathematical model for the aortic excessive, swelling, inflammations,

stretching etc. of a human body muscle by varying the mechanical properties as space and time dependent. In other words these studies have wide application in human body muscle.

These studies have been widely used in seismology, earth quake and civil engineering acoustics, optics, bio-mechanics, applied physics and applied mathematics and also for detecting impurities and food products like cheese, vegetable ghee, human body muscles, bone which are known for their elastic and viscoelastic properties. Viscoelasticity of bones may lead to an understanding or remodeling process and mechanism of bone tissues.

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